Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.; MARTIN SPINX, Wilmington, O.; F. R. HONEY, Ph. B., New Haven, Conn.; ALBERT J. GIBBS, Salida, Col.; and AMELIA BACH, Salida, Col.

When the pursuers met the express they had been in pursuit 8 hours. When the express met the criminal, the pursuers had been following the criminal $8-2\frac{2}{5}=5\frac{3}{5}$ hours, and the criminal had been escaping for $10+5\frac{3}{5}=15\frac{3}{5}$ hours.

As the express and the pursuers traveled at the same rate, the distance traveled by the criminal in $15\frac{3}{5}$ hours was traveled by the pursuers in $8+2\frac{2}{5}=10\frac{2}{5}$ hours. The pursuers, in this time, gained $10\frac{2}{5}\times3=31\frac{1}{5}$ miles. This distance was evidently traveled by the criminal in $15\frac{3}{5}-10\frac{2}{5}=5\frac{1}{5}$ hours.

$\therefore$ The criminal's rate of travel was $31\frac{1}{5}\div5\frac{1}{5}=6$ miles per hour.

The crimiual therefore had the start of $10\times6=60$ miles.

But the pursuers gained 3 miles per hour. Then, to gain 1 mile they had to travel $\frac{1}{3}$ hour, and to gain the 60 miles they had to travel $60\times\frac{1}{3}=20$ hours$=$ the time required.

Also solved by *J. H. DRUMMOND, WILL RYAN, D. G. DORRANCE, Jr., W. H. DRANE, G. B. M. ZERR, FREMONT CRANE, M. E. GRABER, B. F. YANNEY*, and *J. A. MOORE*.

ALGEBRA.

81. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

Show that $$\frac{a_1{}^r}{(a_1-a_2)(a_1-a_3)(a_1-a_4)\ldots(a_1-a_n)}$$

$$+\frac{a_2{}^r}{(a_2-a_1)(a_2-a_3)\ldots(a_2-a_n)}+\ldots\frac{a_n{}^r}{(a_n-a_1)(a_n-a_2)\ldots(a_n-a_{n-1})}$$

is zero if r is less than $n-1$; to 1 if $r=n-1$, and to $a_1+a_2+a_3+\ldots a_n$ if $r=n$.

[*C. Smith's Treatise on Algebra*, Ex. 53, page 104.]

Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, University, Miss.

The fractions being reduced to their least common denominator, every term of the numerator contains the factor a_1-a_2 except the first and the second. If, in the numerator, we put $a_1=a_2$, the first two terms become the same with opposite signs and each of the remaining terms has a zero-factor. Hence the numerator vanishes under this supposition, and, therefore, a_1-a_2 is a factor of it. Similarly every factor of the denominator may be shown to be a factor of the numerator. Now the latter is a homogeneous expression of a degree less than that of the denominator by $n-1-r$, there being $n-1$ factors in the denominator of each of the original fractions.

If $r<n-1$, the numerator is of lower degree than the denominator. But, as proved above, there are as many conditions that cause the numerator to vanish as there are factors in the denominator. In this case the number of these is greater than the degree of the numerator, which is, therefore, identically equal to zero.

If $r=n-1$, the numerator and the denominator are of equal degree, and, being composed of the same factors, the fraction equals 1.

If $r=n$, the degree of the numerator is one greater than that of the denominator. Hence, besides the factors common to both, there must be in the numerator one other factor of the first degree. Since this factor must be symmetrical with reference to a_1, a_2, a_3, etc., it is $a_1+a_2+a_3+\ldots a_n$.

This last is, therefore, the value of the fraction, the numerical coefficient independent of a_1, a_2, a_3, etc., evidently being unity.

Also solved by *C. W. M. BLACK.*

82. Proposed by B. F. YANNEY, A. M., Mount Union College, Alliance, Ohio, and F. P. MATZ, D. Sc., Ph.D. Mechanicsburg, Pa.

$$\left.\begin{array}{l} y^2+yz+z^2=a^2 \\ z^2+zx+x^2=b^2 \\ x^2+xy+y^2=c^2 \end{array}\right\}\text{find } x,\ y, \text{ and } z.$$

[*C. Smith's Treatise on Algebra*, Ex. 31, page 172.]

I. Solution by P. S. BERG, A. M., Principal of Schools, Larimore, N. D.

From (1)+(2)+(3), $2(y^2+z^2+x^2)+yz+zx+xy=a^2+b^2+c^2\ldots\ldots\ldots(4)$.

Squaring (4)

$4(y^2+z^2+x^2)^2+4(y^2+z^2+x^2)(yz+zx+xy)+(yz+zx+xy)^2=(a^2+b^2+c^2)^2\ \ldots(5)$.

From $2(1)^2+2(2)^2+2(3)$

$4(y^2+z^2+x^2)^2+4(y^2+z^2+x^2)(yz+zx+xy)-2(yz+zx+xy)^2=2(a^4+b^4+c^2)\ldots(6)$.

From $\surd\{[(5)-(6)]/3\}$

$yz+zx+xy=\pm\surd\{\frac{1}{3}[(2a^2b^2+2b^2c^2+2a^2c^2)-(a^4+b^4+c^4)]\}\ \ldots\ldots\ldots\ \ldots\ldots(7)$.

Put second member$=m$; then from $\surd[6(7)+2(4)]$

$2(y+z+x)=\pm\surd[2(a^2+b^2+c^2)+6m]\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$.

From $(4)+(7)-2(2)$ $\ 2y(y+z+x)=a^2-b^2+c^2+m.\ \ldots\ldots\ldots\ldots\ldots(9)$.

From $(9)\div(8)$ and restoring m

$$y=\frac{a^2-b^2+c^2\pm\frac{1}{3}\surd[12a^2b^2-3(a^2+b^2-c^2)^2]}{\pm\surd\{2(a^2+b^2+c^2)\pm2\surd[12a^2b^2-3(a^2+b^2-c^2)^2]\}}.$$

Similarly, $$z=\frac{a^2+b^2-c^2\pm\frac{1}{3}\surd[12a^2b^2-3(a^2+b^2-c^2)^2]}{\pm\surd\{2(a^2+b^2+c^2)\pm2\surd[12a^2b^2-3(a^2+b^2+c^2)^2]\}},$$

and $$x=\frac{b^2+c^2-a^2\pm\frac{1}{3}\surd[12a^2b^2-3(a^2+b^2-c^2)^2]}{\pm\surd\{2(a^2+b^2+c^2)\pm2\surd[12a^2b^2-3(a^2+b^2-c^2)]\}}.$$

II. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Subtracting (1) from (2) we get $(x-y)(x+y+z)=b^2-a^2$, or putting $x+y+z=s$, $(x-y)s=b^2-a^2\ldots\ldots(4)$, and subtracting (1) from (3), we thus get $(x-z)s=c^2-a^2\ldots.(5)$. From (4) and (5) we obtain $y=(sx+a^2-b^2)/s\ldots.(6)$, and $z=(sx+a^2-c^2)/s\ldots\ldots(7)$. Adding x to both members of (6) and (7), we have $x+y+z=(3sx+2a^2-b^2-c^2)/s$, or $s^2=3sx+2a^2-b^2-c^2\ldots\ldots(8)$, whence